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HIGHER ALGEBRA.

Abstract.

This course covers several relevant topics from Jacob Lurie's book *Higher Algebra* [1]. We begin by recalling the necessary notions from higher *Higher Topos Theory* [2] that enable us to study algebraic and module objects. Once these notions are in place, we discuss one of the central results of the book, namely the Barr–Beck–Lurie theorem. The course concludes with applications of Barr–Beck–Lurie to proving descent theorems for derived categories.
The course is divided into three parts as follows:

1. **Fibrations, Limits, and Adjoints in Higher Category Theory:** We cover various notions of fibrations (right, left, inner, and Cartesian), adjunctions, and limits and colimits, including Kan extensions.

2. **Operads, Algebra Objects, and Module Objects:** We introduce the theory of operads, which plays a central role in defining algebra and module objects. In particular, we discuss operads such as $\mathbf{Assoc}^{\otimes}$, $\mathbf{Pf}^{\otimes}$, $\mathbf{LM}^{\otimes}$, and related variants, and use them to give precise definitions of these notions. We also study the behavior of limits and colimits of such objects and briefly sketch the Lurie tensor product on presentable stable ∞-categories.

3. **Barr–Beck–Lurie and Its Applications:** The final part of the course is devoted to the higher-categorical version of the Barr–Beck theorem. This theorem can be viewed as a categorical formulation of faithfully flat descent in the language of monads. After stating the Barr–Beck–Lurie theorem, we apply it to prove descent theorems for derived ∞-categories.

[1] Jacob Lurie. Higher algebra

[2] Jacob Lurie. *Higher topos theory*, volume 170 of *Annals of Mathematics Studies*. Princeton University Press, Princeton, NJ, 2009

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[1] Jacob Lurie. Higher algebra.

[2] Jacob Lurie. *Higher topos theory*, volume 170 of *Annals of Mathematics Studies*. Princeton University Press, Princeton, NJ, 2009.